Decision Dependent Uncertainty with Uncertain Reduction

Kartikey Sharma and Omid Nohadani

Northwestern University
Industrial Engineering and Management Sciences

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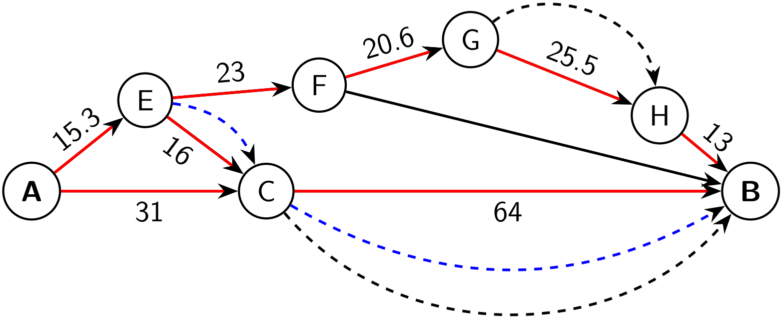
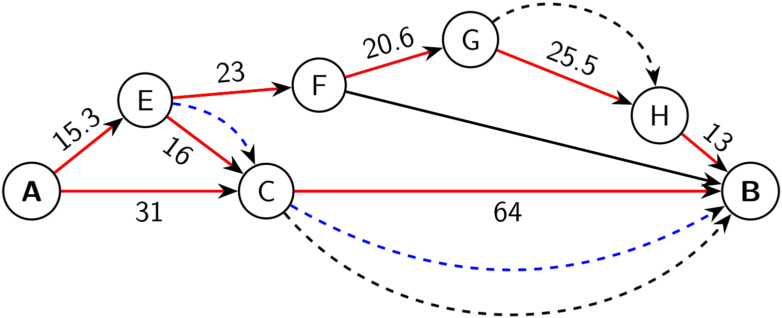


$\Gamma = 1, \quad \gamma = 0.8$

SP	**Nominal** $= 95$	**Worst Case** $= 127$
RSP	**Nominal** $= 97.4$	**Worst Case** $= 110.15$
DDRSP	**Nominal** $= 95.3 + c$	**Worst Case** $= 108.1 + c$

RO with Decision Dependent Uncertainty



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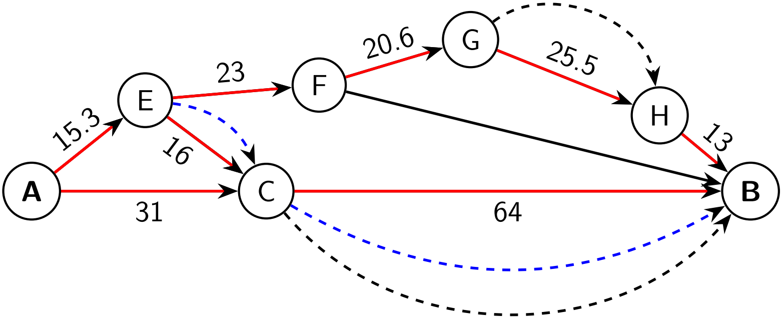

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DDRSP **Nominal** $= 95.3 +$ c **Worst Case** $= 108.1 +$ c

If $\gamma = 0.8$ the c $\geq$ 2.05 makes reduction not useful which also affects the path chosen.

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- Facility location problems
  - The impact of facility on demand may be uncertain
- Information based interpretation
  - If the decision being made is the collection of additional information its affect are uncertain by definition
- Reduction based interpretation
  - The amount of reduction is uncertain

Standard Uncertainty Set

$$\mathcal{U} = \left\{ \mathbf{d} : \sum_{j \in N} \frac{|d_j - \bar{d}_j|}{\hat{d}_j} \leq \Gamma \sqrt{N},\ d_j \in [\bar{d}_j - \Gamma \hat{d}_j, \bar{d}_j + \Gamma \hat{d}_j]\ \forall j \in \mathcal{N} \right\}$$

Mathematical Model

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Decision Dependent Uncertainty Set

$$\mathcal{U}(\mathbf{y}, \gamma) = \left\{ \mathbf{d} : \sum_{j \in N} \frac{|d_j - \bar{d}_j|}{\hat{d}_j} \leq \Gamma\sqrt{N} \right.$$

$$\left. d_j \in [\bar{d}_j - \Gamma\hat{d}_j, \bar{d}_j + (\Gamma - \gamma_j y_j)\hat{d}_j] \right\}$$

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γ_j : reduction parameter

z_j : reduction decision

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How do we choose $\boldsymbol{\gamma}$?

DESCRIBING γ

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Set based information $\boldsymbol{\gamma} \in G$

- Interval Sets:

$$G = \{\boldsymbol{\gamma} : \underline{\boldsymbol{\gamma}} \leq \boldsymbol{\gamma} \leq \overline{\boldsymbol{\gamma}}\}$$

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Distributional uncertainty $\boldsymbol{\gamma} \sim G$

- Finite distribution:

$$G : P[\boldsymbol{\gamma} = \boldsymbol{\gamma}_s] = \pi_s \;\; \forall s = 1, \ldots, N_s$$

Set based information: Robust formulation

$$\min_{\mathbf{x}\in X,\mathbf{y}\in Y}\left\{\mathbf{b}^\top\mathbf{x}+\mathbf{r}^\top\mathbf{y}+\max_{\mathbf{d}\in\mathcal{U}(\mathbf{y},\boldsymbol{\gamma})}\min_{\mathbf{z}\in\Omega(\mathbf{x},\mathbf{d})}\mathbf{c}^\top\mathbf{z}\right\}$$

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- Leads to expanded uncertainty set $\mathcal{U}(\mathbf{y})=\bigcup_{\boldsymbol{\gamma}\in G}\mathcal{U}(\mathbf{y},\boldsymbol{\gamma})$

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- Solve using an affine policy approach

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- Uncertain reduction described set of scenarios of size N_s

$$\min_{\mathbf{x}\in X,\mathbf{y}\in Y} \left\{ \mathbf{b}^\top \mathbf{x} + \mathbf{r}^\top \mathbf{y} + \frac{1}{N_s} \sum_{s=1}^{N_s} \pi_s \left[\max_{\mathbf{d}\in\mathcal{U}(\mathbf{y},\boldsymbol{\gamma}_s)} \min_{\mathbf{z}\in\Omega(\mathbf{x},\mathbf{d})} \mathbf{c}^\top \mathbf{z} \right] \right\}$$

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- Solve using an affine policy approach
- Leads to separate policy for each scenario

Unit Commitment

Day ahead unit commitment with uncertain loads

- 15 buses
- $N_g = 9$ generators (index i)
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The amount of maximum load reduced is uncertain.

Implementation

Load Uncertainty set

$$\mathcal{U}(z,\gamma) = \left\{ \sum_{j \in N} \frac{|d_j - \bar{d}_j|}{\hat{d}_j} \leq \Gamma\sqrt{N}, \;\; d_j \in [\bar{d}_j - \Gamma\hat{d}_j, \bar{d}_j + (\Gamma - \gamma\, z)\hat{d}_j] \right\}$$

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- Only one reduction parameter γ and decision z
- $\Gamma = 2$
- Price of Reduction = 10/load/time
- $\hat{d}_j = 10\%$ of $\bar{d}_j$

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The unit commitment problem uses the following

- Unit commitment model from Lorca et al. [1]
- Decision dependent uncertainty model from Nohadani and Sharma [2]

Solution

- Second stage power generation decisions modeled as affine functions of the realized load given by $z_{it} = w_{it} + W_{it} \sum_{j \in N_d} d_{jt}$

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- Second stage power generation decisions modeled as affine functions of the realized load given by $z_{it} = w_{it} + W_{it} \sum_{j \in N_d} d_{jt}$
- Constraints involving load are
  1. Demand Satisfaction constraints
  2. Capacity constraints
  3. Ramping constraints
  4. Line flow constraints

Power Balance Constraints

Proposition 3 [Lorca et al. [1]]

$$\sum_{i \in N_g} w_{it} = 0, \quad \sum_{i \in N_g} W_{it} = 1 \;\; \forall t \in T$$

Proposition 3 reformulates the power balance constraints as described above. It can be extended to decision dependent uncertainty sets.

Capacity Constraints

Capacity constraints are of the following general form

$$c_{it} \sum_{j \in N_d} d_{jt} \leq h_{it} \; \forall \mathbf{d} \in \mathcal{U}(y)$$

We use the results from Nohadani and Sharma [2] to write the robust problem.

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$$
\begin{aligned}
c_{it} \sum_{j=1}^{N_d} (\bar{d}_{jt} - \Gamma \hat{d}_{jt}) + \max_{\boldsymbol{\chi}, \boldsymbol{\zeta}} \; & \sum_{j=1}^{N_d} ((c_{it} - Mz)\chi_{jt} + c_{it}\zeta_{jt}) \\
\text{s.t.} \; & \sum_{j=1}^{N_d} \frac{|\chi_{jt} + \zeta_{jt} - \Gamma \hat{d}_{jt}|}{\hat{d}_{jt}} \leq \Gamma \sqrt{N_d} \\
& 0 \leq \zeta_{jt} \leq 2\Gamma \hat{d}_{jt} - \hat{d}_{jt}\gamma_{jt} \;\; \forall j \\
& 0 \leq \chi_{jt} \leq \hat{d}_{jt}\gamma_{jt} \;\; \forall j
\end{aligned}
$$

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- z can be either be 0 or 1.
- c_{it} can be $>= 0$ or < 0.
- Leads to 4 possible combinations. The worst case scenarios can be pre-computed the the corresponding cuts generated without iterative cut generation.

Ramping and Line Flow Constraints

In the current model the ramping and line flow constraints are generated using the cut generation algorithm similar to the capacity constraints.

Results

Model	G	Reduced	Obj	Change

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Nominal			103926.4	
Robust	0		115516.2	11.15%

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DDU Fixed	0.5	Yes	114557.5	10.23%
DDU Robust 1	[0.4, 0.6]	Yes	115216.6	10.86%
DDU Robust 2	[0.3, 0.7]	No	115516.2	11.15%

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Robust	0		115516.2	11.15%
DDU Fixed	0.5	Yes	114557.5	10.23%
DDU Robust 1	[0.4, 0.6]	Yes	115216.6	10.86%
DDU Robust 2	[0.3, 0.7]	No	115516.2	11.15%
DDU Stoch. 1	$\{0.4, 0.5, 0.6\}$	Yes	114371.3	10.05%
DDU Stoch. 2	$\{0.3, 0.5, 0.7\}$	Yes	114083.3	9.77%

Conclusions

- Results illustrate how the problem of uncertain reductions can be tackled
- Using cut generation algorithms and exploiting the structure of uncertainty sets provides scalability for both Robust and Stochastic setting.
- Our goal in the future is to solve the problem for larger dimensions.

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kartikeysharma2014@u.northwestern.edu

References

[1] Álvaro Lorca, X Andy Sun, Eugene Litvinov, and Tongxin Zheng. Multistage adaptive robust optimization for the unit commitment problem. *Operations Research*, 64(1):32–51, 2016.

[2] Omid Nohadani and Kartikey Sharma. Optimization under decision-dependent uncertainty. *SIAM Journal on Optimization*, 28(2):1773–1795, 2018.